

Cosmic Jerk and Snap in Penrose's CCC model

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Abstract

We obtain a constraint on cosmological scalars for the FRW metric with a pure radiation fluid source and positive cosmological constant. We demonstrate that this constraint is conformally invariant in the context of Penrose's Conformal Cyclic Cosmology proposal, where the metrics of the late stages of the previous aeon and the early stages of the present aeon are described by FRW cosmologies.

Conformal Cyclic Cosmology (CCC) of Penrose is a cosmological model which postulates the existence of an infinite sequence of aeons, and provides a radical alternative to Inflationary Cosmology [4]. The aeon (\hat{M}, \hat{g}) (which we shall call the present aeon) starts on an initial Big-Bang singularity Σ , which is identified with a space-like future null infinity of the exponentially expanding aeon (\hat{M}, \hat{g}) (from now on called the previous aeon). The cosmological constant of all aeons is assumed to be positive. The bridging space-time $M = \hat{M} \cup \check{M} \cup \Sigma$ is equipped with a regular Lorentzian metric g such that

$$\hat{g} = \hat{\Omega}^2 g, \quad \check{g} = \check{\Omega}^2 g, \quad \text{and} \quad \Sigma = \{\hat{\Omega}^{-1} = 0\} = \{\check{\Omega} = 0\}.$$

The conformal factors satisfy the reciprocal hypothesis $\hat{\Omega} = -\check{\Omega}^{-1}$, and are determined by a Yamabe-type equation on the (M, g) background. The Big-Bang three-surface Σ is singular in the present aeon, but this singularity is only manifest in the conformal factor $\check{\Omega}$. The Weyl curvatures of g, \hat{g}, \check{g} are all equal and are assumed to be finite at Σ . Penrose argues that this is in agreement with the second law of thermodynamics which requires the initial gravitational entropy to be low. This is to be contrasted with a final black hole singularity (of say the Schwarzschild solution) where the Weyl tensor blows up.

We shall assume that the past aeon was described by an FRW metric

$$\hat{g} = -d\hat{t}^2 + \hat{a}^2 h \tag{1}$$

with a pure radiation fluid source and positive cosmological constant. Here $\hat{a} = \hat{a}(\hat{t})$ is the scale factor, and h is a metric on H^3, \mathbb{R}^3 or S^3 with constant curvature $\hat{k} = -1, 0$ or 1 . To the first order in time, the scale factor of this aeon is measured by the Hubble scalar with a dimension of inverse of time. The next three terms in the Taylor expansion of the red-shift are given by dimensionless scalars called deceleration, jerk and snap respectively

$$\hat{q} = -\hat{a} \left(\frac{d\hat{a}}{d\hat{t}} \right)^{-2} \frac{d^2 \hat{a}}{d\hat{t}^2}, \quad \hat{Q} = \hat{a}^2 \left(\frac{d\hat{a}}{d\hat{t}} \right)^{-3} \frac{d^3 \hat{a}}{d\hat{t}^3}, \quad \hat{X} = \hat{a}^3 \left(\frac{d\hat{a}}{d\hat{t}} \right)^{-4} \frac{d^4 \hat{a}}{d\hat{t}^4}. \tag{2}$$

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The Einstein equations with cosmological constant $\hat{\Lambda}$ reduce to the Friedmann equation

$$\left(\frac{d\hat{a}}{d\hat{t}}\right)^2 + \hat{k} = \frac{8\pi G}{3}\hat{\rho}\hat{a}^2 + \frac{\hat{\Lambda}}{3}\hat{a}^2, \quad (3)$$

with the energy-momentum conservation equations giving rise to the density $\hat{\rho} = m\hat{a}^{-4}$.

The Friedmann equation can be reinterpreted as an algebraic constraint between the scalars (2). The general procedure for obtaining such constraints for any matter model has been described in [1], and some special cases were discussed in [2, 7]. This links the measurement of the scalars (2) to a test of General Relativity, or any of its modifications in the spirit of [5]. If one assumes that Einstein equations hold, then measuring the cosmological scalars could determine the equation of state relating the energy density and the momentum in the perfect fluid energy momentum tensor.

To derive the constraint consider a system of three equations consisting of (3) and its first two time derivatives. We regard this as a system of algebraic equations for the constants $(\hat{k}, \hat{\Lambda}, m)$ which can therefore be expressed as functions of $(\hat{a}, \dot{\hat{a}}, \ddot{\hat{a}}, \dddot{\hat{a}})$. Take the third derivative of (3) and substitute the expressions for $(\hat{k}, \hat{\Lambda}, m)$. This yields

$$\hat{X} + 3(\hat{q} + \hat{Q}) + \hat{q}\hat{Q} = 0. \quad (4)$$

This fourth order ODE is equivalent to the Friedmann equation and has an advantage that it appears as a constraint on directly measurable quantities (2).

According to Tod [6], the metric \check{g} in the present aeon is conformally related to the FRW metric \hat{g} in the past aeon by

$$\check{g} = \alpha \hat{a}^{-4} \hat{g} = -d\check{t}^2 + \check{a}^2 h,$$

where $\alpha > 0$ is a constant. As both metrics are of the FRW form, we can compute the conformal transformation properties of the cosmic deceleration, jerk and snap respectively:

$$\check{q} = -\hat{q}, \quad \check{Q} = \hat{Q} - 2\hat{q}, \quad \check{X} = -\hat{X} - 6\hat{Q} + 6\hat{q} - 2\hat{q}^2.$$

Therefore the cosmological scalars are not conformally invariant. We nevertheless explicitly verify that the constraint (4) is conformally invariant up to an overall sign: it holds both in the past and the current aeons.

Our findings agree with the calculation of Tod [6] and Newman [3], who showed that both aeons are diffeomorphic if either of them is described by pure radiation FRW cosmology. In [6] Tod uses the FRW example to motivate a general prescription of finding a conformal factor relating two non-conformally flat aeons, where the past aeon described by a Bianchi type cosmological model. It would be interesting to extend our procedure to this case, where the cosmological scalars (2) presumably need to be constructed out of the total volume element of the spatial slice.

References

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